

UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 1ST FEBRUARY 2001

Organised by the **United Kingdom Mathematics Trust**
from the **School of Mathematics, University of Leeds**



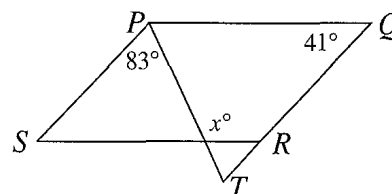
RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 11 or below.
Candidates in Scotland must be in S4 or below.
Candidates in Northern Ireland must be in School Year 12 or below.
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.
Six marks are awarded for each correct answer to Questions 16-25.
Each incorrect answer to Questions 16-20 loses 1 mark.
Each incorrect answer to Questions 21-25 loses 2 marks.
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options**. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. The UK IMC is about solving interesting problems, not about lucky guessing.

The UKMT is a registered charity

- Between which of the following pairs of numbers is there the greatest difference?
 A $-3, 8$ B $-5, -13$ C $1, 11$ D $4, -5$ E $-6, -15$
- A tourist attraction, which opens every day, needs 30 000 visitors per day on average to break even. Last week there were 120 000 visitors. What is the number of visitors needed this week to break even over the two-week period?
 A 43 000 B 90 000 C 180 000 D 210 000 E 300 000
- Which of the following is midway between $\frac{1}{4}$ and $\frac{1}{8}$?
 A $\frac{3}{16}$ B $\frac{1}{6}$ C $\frac{5}{24}$ D $\frac{1}{5}$ E $\frac{7}{32}$
- Old Martha has 5 children, each of whom has 4 children, each of whom has 3 children, each of whom is childless. How many descendants does Old Martha have?
 A 12 B 20 C 25 D 60 E 85

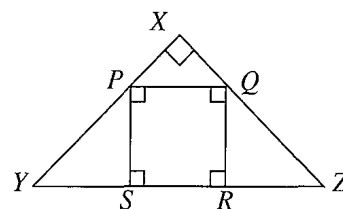
- In the diagram the lines PQ and SR are parallel, as are the lines PS and QT . What is the value of x ?
 A 139 B 138 C 124 D 98 E 97



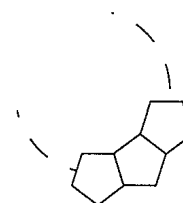
- $2001 = 3 \times 23 \times 29$. Which of the following numbers is also the product of exactly three distinct prime numbers?
 A 45 B 60 C 91 D 105 E 330
- A ream of paper (500 sheets) is 5.4 cm thick. What is the thickness of a single sheet, correct to one significant figure?
 A 1 mm B 0.5 mm C 0.1 mm D 0.05 mm E 0.01 mm

- The diagram shows a right-angled isosceles triangle XYZ which circumscribes a square $PQRS$. The area of triangle XYZ is x . What is the area of square $PQRS$?

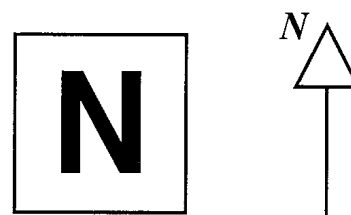
- A $\frac{4x}{9}$ B $\frac{x}{2}$ C $\frac{4x}{5}$ D $\frac{2x}{5}$ E $\frac{2x}{3}$



- Which of the following is the best estimate for the number of seconds which have elapsed since the start of the year 2000?
 A 3×10^4 B 3×10^5 C 3×10^6 D 3×10^7 E 3×10^8
- Equal regular pentagons are placed together to form a ring in the manner shown. The diagram shows the first three pentagons. How many *more* are needed to complete the ring?
 A 6 B 7 C 8 D 9 E 10



11. A square card printed with the letter N is held horizontally, as shown in the diagram, where the arrow indicates the direction of North. The card is turned over by rotating it through 180° about an axis running from East to West, and then turned over by rotating it through 180° about an axis running from North-East to South-West.



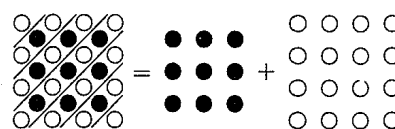
How does the diagram on the card now look to a person facing North?

- A B C D E

12. A long-sleeve shirt has 8 front buttons and 2 cuff buttons; a short-sleeve shirt has 6 front buttons and no cuff buttons. The factory which makes 'Slimboy Shirts' uses 10 times as many front buttons as cuff buttons. What is the ratio of long-sleeve shirts to short-sleeve shirts produced by the factory?

- A 4 : 1 B 4 : 3 C 2 : 3 D 3 : 4 E 1 : 2

13. The diagram shows that $1 + 3 + 5 + 7 + 5 + 3 + 1 = 3^2 + 4^2$.
What is $1 + 3 + 5 + \dots + 1999 + 2001 + 1999 + \dots + 5 + 3 + 1$?



- A $999^2 + 1000^2$ B $1000^2 + 1001^2$ C $1999^2 + 2000^2$
D $2000^2 + 2001^2$ E none of these

14. A square patchwork quilt is made by joining four square pieces of cloth like this , each piece coloured grey and white as shown. Only edges of the same colour are sewn together. How many different quilt patterns are possible? (Two patterns are considered to be the same if one can be rotated to look exactly like the other.)

- A 3 B 4 C 5 D 6 E 8

15. How many of the numbers

$$3\sqrt{11} \quad 4\sqrt{7} \quad 5\sqrt{5} \quad 6\sqrt{3} \quad 7\sqrt{2}$$

are greater than 10?

- A 1 B 2 C 3 D 4 E 5

16. The Pythagoras Patisserie sells triangular cakes at 39p each and square buns at 23p each. For her party, Helen spent exactly £5.12 on an assortment of these cakes and buns. How many items in total did she buy?

- A 15 B 16 C 17 D 18 E 19

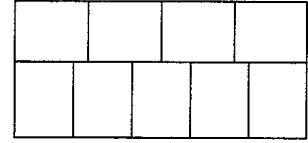
17. Albert Einstein was standing on the station platform thinking about relativity when he noticed that he could see two station clocks. Each clock was digital, showing only hours and minutes. He observed that the display on one clock changed to the next minute 10 seconds before the correct time, whereas the display on the other clock changed to the next minute 10 seconds after the correct time. For what fraction of the day did the clocks show the same time?

- A $\frac{1}{6}$ B $\frac{1}{3}$ C $\frac{2}{3}$ D $\frac{5}{6}$ E $\frac{7}{6}$

18. An athlete covers three consecutive miles by walking the first mile, running the second and cycling the third. He runs twice as fast as he walks, and he cycles one and a half times as fast as he runs. He takes ten minutes longer than he would do if he cycled the three miles. How long does he take by walking, running and cycling?

- A 60 min B 42 min C 36 min D 30 min E 22 min

19. The diagram shows a large rectangle composed of nine identical smaller rectangles. Both the length and breadth of each of these smaller rectangles are whole numbers of centimetres. Which of the following could be the area of the large rectangle?



- A 450 cm² B 630 cm² C 1260 cm² D 1440 cm² E 1620 cm²

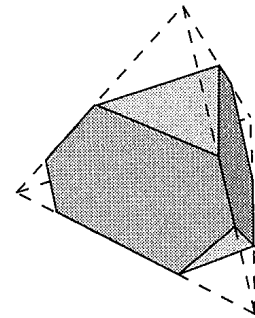
20. In how many different ways can seven different numbers be chosen from the numbers 1 to 9 inclusive so that the seven numbers have a total which is a multiple of 3?

- A fewer than 10 B 10 C 11 D 12 E more than 12

21. Given that x is positive and less than 1, which of the following numbers is the largest?

- A $x^2 + x$ B x^2 C x^3 D $x^3 + x^2$ E x^4

22. A regular tetrahedron with edges of length 6 cm has each corner cut off to produce the solid shown. The triangular faces are all equilateral triangles, but not necessarily all the same size. What is the total length of the edges of the resulting solid?



- A 28 cm B 30 cm C 36 cm
D 48 cm E more information needed

23. For how many values of n are both n and $\frac{n+3}{n-1}$ integers?

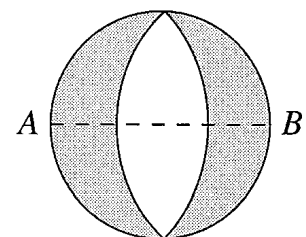
- A 7 B 6 C 4 D 3 E 0

24. A 4 by 4 'antimagic square' is an arrangement of the numbers 1 to 16 inclusive in a square, so that the totals of each of the four rows and four columns and two main diagonals are ten consecutive numbers in some order. The diagram shows an incomplete antimagic square. When it is completed, what number will replace the asterisk?

4	5	7	14
6	13	3	*
11	12	9	
10			

- A 1 B 2 C 8 D 15 E 16

25. AB is a diameter of a circle of radius 1 cm. Two circular arcs of equal radius are drawn with centres A and B . These arcs meet on the circle, as shown. What is the shaded area?



- A $\frac{\pi}{2}$ cm² B 1 cm² C $(\pi - 1)$ cm² D 2 cm² E $\frac{2\pi}{3}$ cm²